The `kinner()` function in the `stokes` package

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```
kinner
```

```
## function (o1, o2, M)
## {
##     stopifnot(arity(o1) == arity(o2))
##     k <- arity(o1)
##     if (missing(M)) {
##         M <- diag(nrow = max(c(index(o1), index(o2))))
##     }
##     out <- 0
##     k <- arity(o1)
##     c1 <- elements(coeffs(o1))
##     c2 <- elements(coeffs(o2))
##     for (no1 in seq_len(nterms(o1))) {
##         for (no2 in seq_len(nterms(o2))) {
##             MM <- matrix(0, k, k)
##             for (i in seq_len(k)) {
##                 for (j in seq_len(k)) {
##                   MM[i, j] <- MM[i, j] + M[index(o1)[no1, i],
##                     index(o2)[no2, j]]
##                 }
##             }
##             out <- out + det(MM) * c1[no1] * c2[no2]
##         }
##     }
##     return(out)
## }
```

To cite the `stokes` package in publications, please use Hankin (2022). Given two k-forms α, β, function `kinner()` returns an inner product $\langle\cdot,\cdot\rangle$ of α and β. If $\alpha = \alpha_1 \wedge\cdots\wedge \alpha_k$ and $\beta = \beta_1 \wedge\cdots\wedge \beta_k$, and we have an inner product $\langle\alpha_i, \beta_j\rangle$ then

$$\langle\cdot,\cdot\rangle = \det\left(\langle\alpha_i, \beta_j\rangle_{ij}\right)$$

We extend this inner product by bilinearity to the whole of $\Lambda^k(V)$.

Some simple examples

Michael Penn uses a metric of

$$
\begin{array}{c} \\ dt \\ dx \\ dy \\ dz \end{array}
\begin{array}{c}
\begin{array}{cccc} dt & dx & dy & dz \end{array} \\
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}
\end{array}
$$

and shows that

$$
\begin{array}{c|cccccc}
 & dt\wedge dx & dt\wedge dy & dt\wedge dz & dx\wedge dy & dx\wedge dz & dy\wedge dz \\
\hline
dt\wedge dx & -1 & 0 & 0 & 0 & 0 & 0 \\
dt\wedge dy & 0 & -1 & 0 & 0 & 0 & 0 \\
dt\wedge dz & 0 & 0 & -1 & 0 & 0 & 0 \\
dx\wedge dy & 0 & 0 & 0 & 1 & 0 & 0 \\
dx\wedge dz & 0 & 0 & 0 & 0 & 1 & 0 \\
dy\wedge dz & 0 & 0 & 0 & 0 & 0 & 1
\end{array}
$$

so, for example, $\langle dt\wedge dx, dt\wedge dx\rangle = -1$ and $\langle dt\wedge dx, dt\wedge dy\rangle = 0$. We can reproduce this relatively easily in the package as follows. First we need to over-write the default values of `dx`, `dy`, and `dz` (which are defined in three dimensions) and define `dt dx dy dz`:

```
dt <- d(1)
dx <- d(2)
dy <- d(3)
dz <- d(4)
p <- c("dt^dx","dt^dy","dt^dz","dx^dy","dx^dz","dy^dz")

mink <- diag(c(1,-1,-1,-1)) # Minkowski metric

M <- matrix(NA,6,6)
rownames(M) <- p
colnames(M) <- p

do <- function(x){eval(parse(text=x))}
for(i in seq_len(6)){
  for(j in seq_len(6)){
    M[i,j] <- kinner(do(p[i]),do(p[j]),M=mink)
  }
}
M
```

```
##       dt^dx dt^dy dt^dz dx^dy dx^dz dy^dz
## dt^dx    -1     0     0     0     0     0
## dt^dy     0    -1     0     0     0     0
## dt^dz     0     0    -1     0     0     0
## dx^dy     0     0     0     1     0     0
## dx^dz     0     0     0     0     1     0
## dy^dz     0     0     0     0     0     1
```

Slightly slicker:

```
outer(p,p,Vectorize(function(i,j){kinner(do(i),do(j),M=mink)}))
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]   -1    0    0    0    0    0
## [2,]    0   -1    0    0    0    0
## [3,]    0    0   -1    0    0    0
## [4,]    0    0    0    1    0    0
## [5,]    0    0    0    0    1    0
## [6,]    0    0    0    0    0    1
```

Tidyup

It is important to remove the `dt`, `dx`, `dt`, `dx` as created above because they will interfere with the other vignettes:

```
rm(dt,dx,dy,dz)
```

References

Hankin, R. K. S. 2022. "Stokes's Theorem in R." arXiv. https://doi.org/10.48550/ARXIV.2210.17008.