Package 'Ispartition'

April 29, 2025

Type Package

Title Nonparametric Estimation and Inference Procedures using Partitioning-Based Least Squares Regression

Version 0.5

Date 2025-04-29

Description Tools for statistical analysis using partitioning-based least squares regression as described in Cattaneo, Farrell and Feng (2020a, <doi:10.48550/arXiv.1804.04916>) and Cattaneo, Farrell and Feng (2020b, <doi:10.48550/arXiv.1906.00202>): lsprobust() for nonparametric point estimation of regression functions and their derivatives and for robust biascorrected (pointwise and uniform) inference; lspkselect() for data-driven selection of the IMSE-optimal number of knots; lsprobust.plot() for regression plots with robust confidence intervals and confidence bands; lsplincom() for estimation and inference for linear combinations of regression functions from different groups.

Depends R (>= 3.1)

License GPL-2

Encoding UTF-8

Imports ggplot2, pracma, mgcv, combinat, matrixStats, MASS, dplyr

RoxygenNote 7.3.1

NeedsCompilation no

Author Matias D. Cattaneo [aut], Max H. Farrell [aut], Yingjie Feng [aut, cre]

Maintainer Yingjie Feng <fengyingjiepku@gmail.com>

Repository CRAN

Date/Publication 2025-04-29 10:30:06 UTC

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lspartition-package Nonparametric Estimation and Inference using Partitioning-Based Least Squares Regression

Description

This package provides tools for statistical analysis using B-splines, wavelets, and piecewise polynomials as described in Cattaneo, Farrell and Feng (2020a): lsprobust for least squares point estimation with robust bias-corrected pointwise and uniform inference procedures; lspkselect for datadriven procedures for selecting the IMSE-optimal number of partitioning knots; lsprobust.plot for regression plots with robust confidence intervals and confidence bands; lsplincom for estimation and inference for linear combination of regression functions of different groups.

The companion software article, Cattaneo, Farrell and Feng (2020b), provides further implementation details and empirical illustrations.

Author(s)

Matias D. Cattaneo, Princeton University, Princeton, NJ. <cattaneo@princeton.edu>.

Max H. Farrell, University of California, Santa Barbara, CA. <maxhfarrell@ucsb.edu>.

Yingjie Feng (maintainer), Tsinghua University, Beijing, China. <fengyingjiepku@gmail.com>.

References

Cattaneo, M. D., M. H. Farrell, and Y. Feng (2020a): Large Sample Properties of Partitioning-Based Series Estimators. Annals of Statistics, 48(3): 1718-1741, 2020.

Cattaneo, M. D., M. H. Farrell, and Y. Feng (2020b): Ispartition: Partitioning-Based Least Squares Regression. R Journal, 12(1): 172-187, 2020.

lspkselect	Tuning Parameter Selection Procedures for Partitioning-Based Re-
	gression Estimation and Inference

Description

lspkselect implements data-driven procedures to select the Integrated Mean Squared Error (IMSE) optimal number of partitioning knots for partitioning-based least squares regression estimators. Three series methods are supported: B-splines, compactly supported wavelets, and piecewise polynomials. See Cattaneo and Farrell (2013) and Cattaneo, Farrell and Feng (2020a) for complete details.

Companion commands: lsprobust for partitioning-based least squares regression estimation and inference; lsprobust.plot for plotting results; lsplincom for multiple sample estimation and inference.

A detailed introduction to this command is given in Cattaneo, Farrell and Feng (2020b).

For more details, and related Stata and R packages useful for empirical analysis, visit https://sites.google.com/site/nppackages/.

lspkselect

Usage

```
lspkselect(y, x, m = NULL, m.bc = NULL, smooth = NULL, bsmooth = NULL,
deriv = NULL, method = "bs", ktype = "uni", kselect = "imse-dpi",
proj = TRUE, bc = "bc3", vce = "hc2", subset = NULL,
rotnorm = TRUE)
## S3 method for class 'lspkselect'
print(x, ...)
## S3 method for class 'lspkselect'
summary(object, ...)
```

Arguments

У	Outcome variable.
х	Independent variable. A matrix or data frame.
m	Order of basis used in the main regression. Default is m=2.
m.bc	Order of basis used to estimate leading bias. Default is m.bc=m+1.
smooth	Smoothness of B-splines for point estimation. When smooth=s, B-splines have s-order continuous derivatives. Default is smooth=m-2.
bsmooth	Smoothness of B-splines for bias correction. Default is bsmooth=m.bc-2.
deriv	Derivative order of the regression function to be estimated. A vector object of the same length as $ncol(x)$. Default is $deriv=c(0,, 0)$.
method	Type of basis used for expansion. Options are "bs" for B-splines, "wav" for compactly supported wavelets (Cohen, Daubechies and Vial, 1993), and "pp" for piecewise polynomials. Default is method="bs".
ktype	Knot placement. Options are "uni" for evenly spaced knots over the support of x and "qua" for quantile-spaced knots. Default is ktype="uni".
kselect	Method for selecting the number of inner knots used by lspkselect. Options are "imse-rot" for a rule-of-thumb (ROT) implementation of IMSE-optimal number of knots, "imse-dpi" for second generation direct plug-in (DPI) implementation of IMSE-optimal number of knots, and "all" for both. Default is kselect="imse-dpi".
proj	If TRUE, projection of leading approximation error onto the lower-order approx- imating space is included for bias correction (splines and piecewise polynomial only). Default is proj=TRUE.
ЬС	Bias correction method. Options are "bc1" for higher-order-basis bias correc- tion, "bc2" for least squares bias correction, and "bc3" for plug-in bias correc- tion. Defaults are "bc3" for splines and piecewise polynomials and "bc2" for wavelets.
vce	Procedure to compute the heteroskedasticity-consistent (HCk) variance-covariance matrix estimator with plug-in residuals. Options are
	 "hc0" for unweighted residuals (HC0). "hc1" for UC1 weighted

• "hc1" for HC1 weights.

	 "hc2" for HC2 weights. Default.
	• "hc3" for HC3 weights.
subset	Optional rule specifying a subset of observations to be used.
rotnorm	If TRUE, ROT selection is adjusted using normal densities.
	further arguments
object	class lspkselect objects.

Value

ks	A matrix may contain k.rot (IMSE-optimal number of knots for the main regression through ROT implementation), k.bias.rot (IMSE-optimal num- ber of knots for bias correction through ROT implementation), k.dpi (IMSE- optimal number of knots for the main regression through DPI implementation), k.bias.dpi (IMSE-optimal number of knots for bias correction through DPI implementation)
opt	A list containing options passed to the function.

Methods (by generic)

- print(lspkselect): print method for class "lspkselect".
- summary(lspkselect): summary method for class "lspkselect".

Author(s)

Matias D. Cattaneo, Princeton University, Princeton, NJ. <cattaneo@princeton.edu>.

Max H. Farrell, University of California, Santa Barbara, CA. <maxhfarrell@ucsb.edu>.

Yingjie Feng (maintainer), Tsinghua University, Beijing, China. <fengyingjiepku@gmail.com>.

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Cattaneo, M. D., M. H. Farrell, and Y. Feng (2020a): Large Sample Properties of Partitioning-Based Series Estimators. Annals of Statistics, 48(3): 1718-1741, 2020.

Cattaneo, M. D., M. H. Farrell, and Y. Feng (2020b): Ispartition: Partitioning-Based Least Squares Regression. R Journal, 12(1): 172-187, 2020.

Cohen, A., I. Daubechies, and P.Vial (1993): Wavelets on the Interval and Fast Wavelet Transforms. Applied and Computational Harmonic Analysis 1(1): 54-81.

See Also

lsprobust,lsprobust.plot,lsplincom

lsplincom

Examples

```
x <- data.frame(runif(500), runif(500))
y <- sin(4*x[,1])+cos(x[,2])+rnorm(500)
est <- lspkselect(y, x)
summary(est)</pre>
```

lsplincom

Linear Combination of Estimators for Ispartition Package

Description

lsplincom implements user-specified linear combinations across different data sub-groups for regression functions estimation, and computes corresponding (pointwise and uniform) robust biascorrected inference measures. Estimation and inference is implemented using the **lspartition** package. See Cattaneo and Farrell (2013) and Cattaneo, Farrell and Feng (2020a) for complete details.

A detailed introduction to this command is given in Cattaneo, Farrell and Feng (2020b).

For more details, and related Stata and R packages useful for empirical analysis, visit https://sites.google.com/site/nppackages/.

Usage

```
lsplincom(y, x, G, R, eval = NULL, neval = NULL, level = 95,
band = FALSE, cb.method = NULL, cb.grid = NULL, cb.ngrid = 50,
B = 1000, subset = NULL, knot = NULL, ...)
```

S3 method for class 'lsplincom'
print(x, ...)

S3 method for class 'lsplincom'
summary(object, ...)

Arguments

У	Outcome variable.
х	Independent variable. A matrix or data frame.
G	Group indicator. It may take on multiple discrete values.
R	A numeric vector giving the linear combination of interest. Each element is the coefficient of the conditional mean estimator of one group, and they are ordered ascendingly along the value of G.
eval	Evaluation points. A matrix or data frame.
neval	Number of quantile-spaced evaluating points.
level	Confidence level used for confidence intervals; default is level=95.
band	If TRUE, the critical value for constructing confidence band is calculated. Default is band=FALSE.

cb.method	Method used to calculate the critical value for confidence bands. Options are "pl" for a simulation-based plug-in procedure, and "wb" for a wild bootstrap procedure. If band=TRUE with cb.method unspecified, default is cb.method="pl"
cb.grid	A matrix containing all grid points used to construct confidence bands. Each row correponds to the coordinates of one grid point.
cb.ngrid	A numeric vector of the same length as ncol(x). Each element corresponds to the number of grid points for each dimension used to implement uniform inference. Default is uni.ngrid=50.
В	Number of simulated samples used to obtain the critical value for confidence bands. Default is B=1000.
subset	Optional rule specifying a subset of observations to be used.
knot	A list of numeric vectors giving the knot positions (including boundary knots) for each dimension which are used in the main regression. The length of the list is equal to ncol(x). If not specified, it uses the number of knots either specified by users or computed by the companion command lspkselect to generate the corresponding knots according to the rule specified by ktype. See help for lsprobust.
	Arguments to be passed to the function. See lsprobust.
object	class lsplincom objects.

Value

Estimate	A matrix containing eval (grid points), N (effective sample sizes), tau.cl (point estimates with a basis of order m), tau.bc (bias corrected point estimates with a basis of order m.bc), se.cl (standard error corresponding to tau.cl), and se.rb (robust standard error).
sup.cval	Critical value for constructing confidence bands.
opt	A list containing options passed to the function.

Methods (by generic)

- print(lsplincom): print method for class "lsplincom".
- summary(lsplincom): summary method for class "lsplincom"

Author(s)

Matias D. Cattaneo, Princeton University, Princeton, NJ. <cattaneo@princeton.edu>.

Max H. Farrell, University of California, Santa Barbara, CA. <maxhfarrell@ucsb.edu>.

Yingjie Feng (maintainer), Tsinghua University, Beijing, China. <fengyingjiepku@gmail.com>.

References

Cattaneo, M. D., M. H. Farrell, and Y. Feng (2020a): Large Sample Properties of Partitioning-Based Series Estimators. Annals of Statistics, 48(3): 1718-1741, 2020.

Cattaneo, M. D., M. H. Farrell, and Y. Feng (2020b): lspartition: Partitioning-Based Least Squares Regression. R Journal, 12(1): 172-187, 2020.

lsprobust

See Also

lsprobust,lspkselect,lsprobust.plot,

Examples

```
x <- runif(500)
y <- sin(4*x)+rnorm(500)
z <- c(rep(0, 250), rep(1, 250))
est <- lsplincom(y, x, z, c(-1, 1))
summary(est)
```

lsprobust

Partitioning-Based Least Squares Regression with Robust Inference.

Description

1sprobust implements partitioning-based least squares point estimators for the regression function and its derivatives. It also provides robust bias-corrected (pointwise and uniform) inference, including simulation-based confidence bands. Three series methods are supported: B-splines, compact supported wavelets, and piecewise polynomials. See Cattaneo and Farrell (2013) and Cattaneo, Farrell and Feng (2020a) for complete details.

Companion commands: lspkselect for data-driven IMSE-optimal selection of the number of knots on rectangular partitions; lsprobust.plot for plotting results; lsplincom for multiple sample estimation and inference.

A detailed introduction to this command is given in Cattaneo, Farrell and Feng (2020b).

For more details, and related Stata and R packages useful for empirical analysis, visit https://sites.google.com/site/nppackages/.

Usage

```
lsprobust(y, x, eval = NULL, neval = NULL, method = "bs", m = NULL,
  m.bc = NULL, deriv = NULL, smooth = NULL, bsmooth = NULL,
  ktype = "uni", knot = NULL, nknot = NULL, same = TRUE,
  bknot = NULL, bnknot = NULL, J = NULL, bc = "bc3", proj = TRUE,
  kselect = "imse-dpi", vce = "hc2", level = 95, uni.method = NULL,
  uni.grid = NULL, uni.ngrid = 50, uni.out = FALSE, band = FALSE,
  B = 1000, subset = NULL, rotnorm = TRUE)
## S3 method for class 'lsprobust'
print(x, ...)
## S3 method for class 'lsprobust'
summary(object, ...)
```

Arguments

У	Outcome variable.
х	Independent variable. A matrix or data frame.
eval	Evaluation points. A matrix or data frame.
neval	Number of quantile-spaced evaluating points.
method	Type of basis used for expansion. Options are "bs" for B-splines, "wav" for compactly supported wavelets (Cohen, Daubechies and Vial, 1993), and "pp" for piecewise polynomials. Default is method="bs".
m	Order of basis used in the main regression. Default is m=2. For B-splines, if smooth is specified but m is unspecified, default is m=smooth+2.
m.bc	Order of basis used to estimate leading bias. Default is m.bc=m+1. For B-splines, if bsmooth is specified but m.bc is unspecified, default is m.bc=bsmooth+2.
deriv	Derivative order of the regression function to be estimated. A vector object of the same length as $ncol(x)$. Default is $deriv=c(0,, 0)$.
smooth	Smoothness of B-splines for point estimation. When smooth=s, B-splines have s-order continuous derivatives. Default is smooth=m-2.
bsmooth	Smoothness of B-splines for bias correction. Default is bsmooth=m.bc-2.
ktype	Knot placement. Options are "uni" for evenly-spaced knots over the support of x and "qua" for quantile-spaced knots. Default is ktype="uni".
knot	A list of numeric vectors giving the knot positions (including boundary knots) for each dimension which are used in the main regression. The length of the list is equal to $ncol(x)$. If not specified, it uses the number of knots either specified by users or computed by the companion command lspkselect to generate the corresponding knots according to the rule specified by ktype.
nknot	A numeric vector of the same length as $ncol(x)$. Each element corresponds to the number of <i>inner</i> partitioning knots for each dimension used in the main regression. If not specified, nknot is computed by the companion command lspkselect.
same	If TRUE, the same knots are used for bias correction as that for the main regression. Default is same=TRUE.
bknot	A list of numeric vectors giving knot positions used for bias correction. If not specified and same=FALSE, it uses the number of knots either specified by users or computed by the companion command lspkselect to generate knots according to the rule specified by ktype.
bnknot	A numeric vector of the same length as $ncol(x)$. Each element corresponds to the number of <i>inner</i> partitioning knots for each dimension used for bias correction. If not specified, bnknot is computed by the companion command lspkselect.
J	A numeric vector containing resolution levels of father wavelets for each dimension.
bc	Bias correction method. Options are "bc1" for higher-order-basis bias correc- tion, "bc2" for least squares bias correction, and "bc3" for plug-in bias correc- tion. Default are "bc3" for splines and piecewise polynomials and "bc2" for wavelets.

lsprobust

proj	If TRUE, projection of leading approximation error onto the lower-order approxi- mation space is included for bias correction (splines and piecewise polynomials only). Default is proj=TRUE.
kselect	Method for selecting the number of <i>inner</i> knots used by lspkselect. Options are "imse-rot" for ROT implementation of IMSE-optimal number of knots and "imse-dpi" for second generation of DPI implementation of IMSE-optimal number of knots. Default is kselect="imse-dpi".
vce	Procedure to compute the heteroskedasticity-consistent (HCk) variance-covariance matrix estimator with plug-in residuals. Options are
	 "hc0" for unweighted residuals (HC0).
	• "hc1" for HC1 weights.
	 "hc2" for HC2 weights. Default. "hc3" for HC3 weights.
	-
level	Confidence level used for confidence intervals; default is level=95.
uni.method	Method used to implement uniform inference. Options are "pl" for a simulation- based plug-in procedure, "wb" for a wild bootstrap procedure. If unspecified, neither procedure is implemented. Default is uni.method=NULL.
uni.grid	A matrix containing all grid points used to implement uniform inference. Each row correponds to the coordinates of one grid point.
uni.ngrid	A numeric vector of the same length as ncol(x). Each element corresponds to the number of grid points for each dimension used to implement uniform inference. Default is uni.ngrid=50.
uni.out	If TRUE, the quantities used to implement uniform inference is outputted. Default is uni.out=FALSE.
band	If TRUE, the critical value for constructing confidence band is calculated. De- fault is band=FALSE. If band=TRUE with uni.method unspecified, default is uni.method="pl".
В	Number of simulated samples used to obtain the critical value for confidence bands. Default is B=1000.
subset	Optional rule specifying a subset of observations to be used.
rotnorm	If TRUE, ROT selection is adjusted using normal densities.
	further arguments
object	class 1sprobust objects.

Value

Estimate	A matrix containing eval (grid points), N (effective sample sizes), tau.cl (point estimates with a basis of order m), tau.bc (bias corrected point estimates with a basis of order m.bc), se.cl (standard error corresponding to tau.cl), and se.rb (robust standard error).
k.num	A matrix containing the number of inner partitioning knots used in the main regression and bias correction for each covariate.
knot	A list of knots for point estimation.

lsprobust

bknot	A list of knots for bias correction.
sup.cval	Critical value for constructing confidence band.
uni.output	A list containing quantities used to implement uniform inference.
opt	A list containing options passed to the function.

Methods (by generic)

- print(lsprobust): print method for class "lsprobust"
- summary(lsprobust): summary method for class "lsprobust"

Author(s)

Matias D. Cattaneo, Princeton University, Princeton, NJ. <cattaneo@princeton.edu>.

Max H. Farrell, University of California, Santa Barbara, CA. <maxhfarrell@ucsb.edu>.

Yingjie Feng (maintainer), Tsinghua University, Beijing, China. <fengyingjiepku@gmail.com>.

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Cattaneo, M. D., and M. H. Farrell (2013): Optimal convergence rates, Bahadur representation, and asymptotic normality of partitioning estimators. Journal of Econometrics 174(2): 127-143.

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Cattaneo, M. D., M. H. Farrell, and Y. Feng (2020b): Ispartition: Partitioning-Based Least Squares Regression. R Journal, 12(1): 172-187, 2020.

Cohen, A., I. Daubechies, and P.Vial (1993): Wavelets on the Interval and Fast Wavelet Transforms. Applied and Computational Harmonic Analysis 1(1): 54-81.

See Also

lspkselect, lsprobust.plot, lsplincom

Examples

```
x <- data.frame(runif(500), runif(500))
y <- sin(4*x[,1])+cos(x[,2])+rnorm(500)
est <- lsprobust(y, x)
summary(est)</pre>
```

Description

lsprobust.plot plots estimated regression functions and confidence regions using the **lspartition** package. See Cattaneo and Farrell (2013) and Cattaneo, Farrell and Feng (2020a) for complete details.

Companion command: lsprobust for partitioning-based least squares regression estimation and inference; lsprobust.plot for plotting results; lsplincom for multiple sample estimation and inference.

A detailed introduction to this command is given in Cattaneo, Farrell and Feng (2020b).

For more details, and related Stata and R packages useful for empirical analysis, visit https://sites.google.com/site/nppackages/.

Usage

```
lsprobust.plot(..., alpha = NULL, type = NULL, CS = "ci",
CStype = NULL, title = "", xlabel = "", ylabel = "", lty = NULL,
lwd = NULL, lcol = NULL, pty = NULL, pwd = NULL, pcol = NULL,
CSshade = NULL, CScol = NULL, legendTitle = NULL,
legendGroups = NULL)
```

Arguments

	Objects returned by lsprobust.
alpha	Numeric scalar between 0 and 1, the significance level for plotting confidence regions. If more than one is provided, they will be applied to data series accordingly.
type	String, one of "line" (default), "points", "binscatter", "none" or "both", how the point estimates are plotted. If more than one is provided, they will be applied to data series accordingly.
CS	String, type of confidence sets. Options are "ci" for pointwise confidence in- tervals, "cb" for uniform confidence bands, and "all" for both.
CStype	String, one of "region" (shaded region, default), "line" (dashed lines), "ebar" (error bars), "all" (all of the previous) or "none" (no confidence region), how the confidence region should be plotted. If more than one is provided, they will be applied to data series accordingly. If CS = "all", pointwise confidence intervals are forced to be represented by error bars, and uniform bands are represented by both lines and regions.
title	String, title of the plot.
xlabel	Strings, labels for x-axis.
ylabel	Strings, labels for y-axis.

lty	Line type for point estimates, only effective if type is "line" or "both". 1 for solid line, 2 for dashed line, 3 for dotted line. For other options, see the instructions for ggplot2 or par. If more than one is provided, they will be applied to data series accordingly.
lwd	Line width for point estimates, only effective if type is "line" or "both". Should be strictly positive. For other options, see the instructions for ggplot2 or par. If more than one is provided, they will be applied to data series accordingly.
lcol	Line color for point estimates, only effective if type is "line" or "both". 1 for black, 2 for red, 3 for green, 4 for blue. For other options, see the instructions for ggplot2 or par. If more than one is provided, they will be applied to data series accordingly.
pty	Scatter plot type for point estimates, only effective if type is "points" or "both". For options, see the instructions for ggplot2 or par. If more than one is provided, they will be applied to data series accordingly.
pwd	Scatter plot size for point estimates, only effective if type is "points" or "both". Should be strictly positive. If more than one is provided, they will be applied to data series accordingly.
pcol	Scatter plot color for point estimates, only effective if type is "points" or "both". 1 for black, 2 for red, 3 for green, 4 for blue. For other options, see the instructions for ggplot2 or par. If more than one is provided, they will be applied to data series accordingly.
CSshade	Numeric, opaqueness of the confidence region, should be between 0 (transparent) and 1. Default is 0.2. If more than one is provided, they will be applied to data series accordingly.
CScol	Color for confidence region. 1 for black, 2 for red, 3 for green, 4 for blue. For other options, see the instructions for ggplot2 or par. If more than one is provided, they will be applied to data series accordingly.
legendTitle	String, title of legend.
legendGroups	String vector, group names used in legend.

Details

Companion command: 1sprobust for partition-based least-squares regression estimation.

Value

A standard ggplot2 object is returned, hence can be used for further customization.

Author(s)

Matias D. Cattaneo, Princeton University, Princeton, NJ. <cattaneo@princeton.edu>. Max H. Farrell, University of California, Santa Barbara, CA. <maxhfarrell@ucsb.edu>.

Yingjie Feng (maintainer), Tsinghua University, Beijing, China. <fengyingjiepku@gmail.com>.

lsprobust.plot

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Cattaneo, M. D., M. H. Farrell, and Y. Feng (2020b): Ispartition: Partitioning-Based Least Squares Regression. R Journal, 12(1): 172-187, 2020.

See Also

lsprobust, lspkselect, lsplincom, ggplot2.

Examples

x <- runif(500)
y <- sin(4*x)+rnorm(500)
est <- lsprobust(y, x)
lsprobust.plot(est)</pre>

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